

LESSON 2-1

Direct Variation



Lesson Overview

Mathematics Objectives

- ✓ Identify situations that can be modeled by a direct variation.
- ✓ Write equations involving direct variation.
- ✓ Solve equations involving direct variation.

Language Objective

- ✓ Use mathematical language to explain whether a situation or a table of values represents a direct variation.

Essential Understanding

A situation represented by two variables is a direct variation when the ratio of one variable to the other is always equal to a constant of variation.

Previously in this course, students:

- Analyzed linear relationships to write and solve linear equations.
- Wrote and interpreted linear functions that arise in applications.

In this lesson, students:

- Write and solve direct variation equations.
- Use properties of direct variation to solve problems.

Later in this topic, students will:

- Write equations in slope-intercept, point-slope, and standard forms.
- Write equations of lines that are parallel and perpendicular to given lines.

This lesson emphasizes a blend of *conceptual understanding* and *procedural skill*.

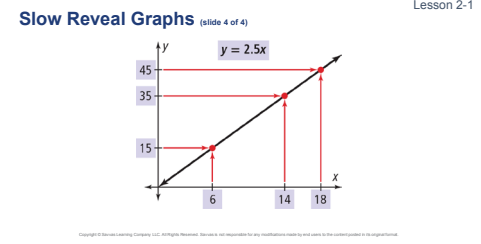
- Students gain a conceptual understanding of direct variation and how to recognize it in descriptions of situations and tables of values.
- Students develop skills in writing and solving direct variation equations.



Presentation is available at SavvasRealize.com.

Math Talk

For use before this lesson.



TEKS A.3C

Slow Reveal Graphs Display and describe the first graph, pointing out the equation of the line. Ask students to suggest ways to determine the x-value of the data point. Students might suggest trial and error, substituting the y-value into the equation, or dividing the y-value by 2.5. Once students have the correct answer, reveal the next graph. Continue until students have all three x-values. Focus on the method of determining the x-values rather than just the answers.

Skills Review & Practice

Present or assign to reactivate prior knowledge.

- **R03** Points on the Coordinate Plane

Vocabulary Builder

Use the Vocabulary Builder activity online to support understanding of the terms *constant of variation* and *direct variation*.



Texas Essential Knowledge and Skills (TEKS)

Linear functions, equations, and inequalities. The student applies the mathematical process standards when using properties of linear functions to write and represent in multiple ways, with and without technology, linear equations, inequalities, and systems of equations. The student is expected to:

A.2D write and solve equations involving direct variation.

Mathematical Process Standards

The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

A.1E create and use representations to organize, record, and communicate mathematical ideas.

A.1F analyze mathematical relationships to connect and communicate mathematical ideas.



English Language Proficiency Standards (ELPS)

LISTENING 1C Follow oral directions with accuracy.



College and Career Readiness Standards

VIII.A.3 Use mathematical language for reasoning, problem solving, making connections, and generalizing.

Standards in **VII** Problem Solving and Reasoning, **VIII** Communication and Representation, and **IX** Connections are infused throughout the program.



EXPLORE & SHARE

INSTRUCTIONAL FOCUS Students analyze mathematical relationships among data in an everyday problem and communicate their ideas.

STUDENT COMPANION Students can do their work for the task on their *Student Companion* pages, which you can print from SavvasRealize.com.

Before WHOLE CLASS

Implement Tasks that Promote Reasoning and Problem Solving

Q: Have you ever compared different types of ticket prices to a sporting event or a community performance? How could you determine the total prices if you went with other people?
[Sample: The cost to attend a play had a discount if five or more people bought tickets together. The cost individually was the number of tickets times the cost. Buying tickets together was that cost minus the discount.]

Q: What are the types of tickets a person could buy to see the baseball game?
[Field Box tickets, Club Level tickets, and View Level tickets]

Q: Does the chart show a discount for buying multiple tickets? How can you tell?
[Sample: The chart shows no discount for multiple tickets. Divide each price by the number of tickets to see that the price per ticket is the same, regardless of the number of tickets.]

During SMALL GROUP

Support Productive Struggle in Learning Mathematics

Q: What variables should you represent by x and y in your equations?
[Sample: Let x represent the cost of the tickets for each location, and let y equal the total cost for each location.]

Q: What is the total number of tickets that will be purchased? What are the two ways they could be bought?
[Sample: There will be 4 tickets. They could be bought individually, or Ken could buy all the tickets.]

For Early Finishers

Q: What equation would represent the total cost for buying 6 of a single type of ticket?
[Sample: Let x represent the cost per ticket, and let y represent the total cost. The equation would be $y = 6x$.]

After WHOLE CLASS

Facilitate Meaningful Mathematical Discourse

Q: Why did you need three equations to relate the number of tickets purchased to the total cost?
[Sample: Each location had a different ticket price.]

Q: If you were to graph each equation, what do you think the graphs would look like? Explain.
[Sample: Each graph would be a straight line because the change in the y value would change by the same amount for each change in x .]

Explore & Share is available at SavvasRealize.com.

STUDENT EDITION

2-1

Direct Variation

I CAN... write and solve equations involving direct variation.

VOCABULARY

- constant of variation
- direct variation

TEKS

A.2D Write and solve equations involving direct variation.

Mathematical Process Standards A.1A, A.1C, A.1E, A.1F, A.1G

EXPLORE & SHARE

Ken wants to see his favorite baseball team play. He found the ticket price chart online.

Location	2 Tickets	5 Tickets
Field Box	\$110	\$275
Club Level	\$165	\$412.50
View Level	\$42	\$105

A. What equations relate the number of tickets purchased to the total cost for each location?

B. Suppose Ken invites 3 friends to go with him to a game. He is buying all of the tickets together, but each friend will pay him back. What is the total cost for club-level tickets? What amount should each friend pay Ken for a club-level ticket?

C. **Analyze Math Relationships** How can you verify that the cost per ticket is the same no matter how many tickets are purchased?

How do you write and solve equations

SAMPLE RESPONSES

- A. Let x represent the cost per ticket and y represent the total cost. For the Field Box, the equation is $y = 55x$. For the Club Level, the equation is $y = 82.5x$. For the View Level, the equation is $y = 21x$.
- B. Including Ken's ticket, the total cost is \$330.00. Each friend should pay him \$82.50 for a Club Level ticket.
- C. For each type of ticket, divide the cost in the 2 Tickets that column by 2 and divide the cost in the 5 Tickets column by 5. Make sure the quotients are the same.

PROCESS STANDARDS CONNECTION

Use with EXPLORE & SHARE

Communicate Reasoning How did you determine the amount each friend should pay Ken for a Club Level ticket?

[Answers may vary. Sample: First, find the cost of 2 Club Level tickets. Then divide that amount by 2 to determine the amount each friend should pay Ken.]



INTRODUCE THE ESSENTIAL QUESTION

Establish Mathematics Goals to Focus Learning

Students use a constant of variation to write and solve direct variation equations.

EXAMPLE 1 Identify Direct Variation

Build Procedural Fluency From Conceptual Understanding

Q: What is meant in the first sentence by the expression *related variables*?

[Sample: Any change in one of the variables is associated with a change in the other variable.]

Q: Could you have a direct variation situation in which k is negative? Explain.

[Sample: Yes; it would mean that x and y always have opposite signs, and the slope of the line on the graph would be negative.]

Language Routine: Stronger and Clearer Each Time

Help students develop strong math communication skills by having them work in pairs to answer the Process Standards Connection question. Have them take turns answering the question aloud to their partner. Then, have them repeat their answer, trying to make their description more concise. Provide these prompts to help them revise their wording.

Q: What steps could you take to determine whether the data represent a direct variation?

Q: How could you revise your wording to make the meaning clearer?

Q: How could you make the same point by using fewer words?

Examples are available at SavvasRealize.com.

CONCEPTUAL UNDERSTANDING

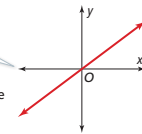
EXAMPLE 1 Identify Direct Variation

How do you identify a direct variation?

Two related variables form a **direct variation** when an increase in one variable implies a proportional increase in the other. You can represent a direct variation algebraically or graphically.

$y = kx$,
with $k \neq 0$

When x increases by 1 unit,
 y increases by k units.



The value k is the **constant of variation**. It is equal to the slope of the line that represents the direct variation. The graph of a direct variation always passes through the origin. When $y = kx$, we say that x and y **vary directly**.

When $x \neq 0$, you can solve for k , which is equal to $\frac{y}{x}$. You can use this to determine whether a table of x and y values represents a direct variation.

COMMON ERROR

Look at all the data. The quotient $\frac{y}{x}$ must be the same for every pair of x and y for the relationship to be a direct variation.

x	2	4	6	9
y	10	20	30	45
$\frac{y}{x}$	5	5	5	5

$\frac{y}{x}$ is constant, so this is a direct variation.
The constant of variation is 5.

x	3	5	10	12
y	18	30	50	60
$\frac{y}{x}$	6	6	5	5

$\frac{y}{x}$ is not constant, so this is not a direct variation.

EXAMPLE 1 CONTINUED ON NEXT PAGE

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PROCESS STANDARDS CONNECTION

Use with
EXAMPLE 1

Select Techniques How can you determine whether the values in a data table represent a direct variation?

[Answers may vary. Sample: Find $\frac{y}{x}$ for the first column of numbers to find a possible value of k . Then multiply each of the remaining x values by that value of k to determine whether you get the corresponding value of y .]

Targeted ELPS Support LISTENING 1C

PRE-PRODUCTION Point to the first data table. Then point to the 10 and 2 as you slowly say, "Divide 10 by 2."

Q: Point to the numbers that you should divide. [Students should point to the 10 and 2.]

Q: Point to the numbers you should divide in the second table. [Students should point to 20 and 4.]

BEGINNING Point to the numbers in the first data table as you slowly say, "Divide 10 by 2."

Q: What should you do first? [Divide 10 by 2.]

Q: Does this match the given answer? [Yes; it is 5.]

Q: What should you do next? [Divide 20 by 4.]

Q: Does this match the given answer? [Yes; it is 5.]

Continue for the other columns.

INTERMEDIATE Point to the first data table. Slowly say, "Divide the y -value by the x -value for each column."

Q: What should you do? [Divide for each column.]

Q: What is the answer for 10 divided by 2? [5]

Continue for the other columns. Then ask:

Q: Is the answer the same or different for the columns? [It is the same.]

HIGH INTERMEDIATE Point to the first data table. Say, "Divide the y -value by the x -value for each column. Then, compare them."

Q: Is y divided by x the same or different for each column? [They are the same. All are 5.]

Repeat for the second data table.

ADVANCED Point to the first data table. Say, "Find k by dividing the y -value by the x -value for each column. Then, compare the answers." Repeat for the second data table."

Q: Did your answer match those shown in the table? [Yes.]

Q: Were the answers the same or different in each table? [The answers were the same in the first table and different in the second.]



Activity

EXAMPLE 1 CONTINUE

Try It! Answers

- 1. a. Not a direct variation; the values of $\frac{y}{x}$ are not constant:
 $\frac{1}{2}, \frac{1}{2}, \frac{3}{8}, \frac{1}{4}, \frac{5}{32}$
- b. Direct variation; the values of $\frac{y}{x}$ are constant:
 $-2, -2, -2, -2, -2$

EXAMPLE 2 Determine Direct Variation

Use and Connect Mathematical Representations

Q: How do you know from the problem description in Part A that you can model the total cost using the equation $y = 3x$?
[Sample: The total cost y increases by 3 for every increase of 1 in the number of blankets. For 1 blanket, the total cost is \$3. For \$2 blankets, the total cost is \$6, and so on.]

Q: What two properties must a graph have to represent a direct variation?
[It must be a straight line and pass through $(0, 0)$.]

Q: How can you use the information in Part B to write a model for the situation?
[Sample: The toy sinks at a constant rate, so the model is linear. The toy is sinking, so the rate of change of its height above the ground is negative, and the coefficient of x is $-\frac{1}{2}$. At time 0 when $x = 0$, the toy is 4 ft above the ground, so the y -intercept is 4. An equation that models this situation is $y = 4 - \frac{1}{2}x$.]

Try It! Answers

- 2. a. Not a direct variation; at time 0, the volume of the ball is not equal to 0. The graph representing the situation does not pass through $(0, 0)$.
- b. Direct variation; at time 0, the height of the plant is equal to 0. The graph that represents the situation passes through $(0, 0)$ and then changes at a constant rate.



Examples are available at SavvasRealize.com.

EXAMPLE 1 CONTINUED

Try It! 1. Determine whether each table of values represents a direct variation.

a.

x	2	4	8	16	32
y	1	2	3	4	5

b.

x	-6	-3	6	9	12
y	12	6	-12	-18	-24

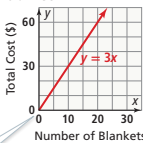
EXAMPLE 2 Determine Direct Variation

Determine whether each situation represents a direct variation.

A. A factory makes blankets. It costs \$3 to make one blanket.

Let x be the number of blankets the factory makes, and let y be the total cost of producing x blankets. You can model the total cost using the equation $y = 3x$.

Because the relationship can be expressed in the form $y = kx$, it is a direct variation.

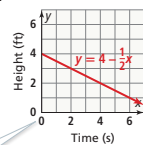


The graph passes through the point $(0, 0)$.

B. A pool toy tossed into a swimming pool starts at water level, 4 ft above the pool bottom, and sinks at a constant rate of $\frac{1}{2}$ ft per second.

Let x be the number of seconds since the toy started to sink, and let y be its height above the bottom. You can model the height of the toy using the equation $y = 4 - \frac{1}{2}x$.

Because the relationship cannot be expressed in the form $y = kx$, it is not a direct variation.



The graph does not pass through the point $(0, 0)$.

Try It! 2. Determine whether each situation is an example of direct variation. Explain your reasoning.

- a. The volume of an inflated ball starts at 314 cm^3 and increases at a constant rate of 12.5 cm^3 per second.
- b. The height of a plant starts at 0 in. above the ground and increases at a constant rate of 0.25 in. per day.

EXIT TICKET

Use this Exit Ticket anytime after Example 2.

ITEM	TEKS	DOK
1	A.2D	1
2	A.2D	2

Use the Exit Ticket results to inform instructional decisions.

Instructional Implications

Question 1 If students chose A or D, they may have assumed that a constant of variation is always positive. As your instruction on direct variation continues, plan to address common errors and misconceptions:

- Reinforce that the constant of variation for a direct variation relationship can be either positive or negative. Use Practice & Problem Solving item 19 if needed.
- Encourage students to recognize that if the constant of variation is negative, x and y will always have opposite signs.



Formative Assessment The Exit Ticket is available at Savvas Realize® and in the Assessment Sourcebook.

Name _____

enVision Algebra 1

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2-1 Exit Ticket

Direct Variation

1. Which direct variation equation relates x and y ?

x	8	12	-32	-36	-48
y	-2	-3	8	9	12

- A $y = \frac{1}{4}x$
- B $y = -\frac{1}{4}x$
- C $y = 4x$
- D $y = -4x$

2. The value of y varies directly with x . When the value of x is -2, the value of y is 3. What is the value of y when x is 18?

- A 12
- B 27
- C -12
- D -27



Activity

EXAMPLE 3 Write a Direct Variation Equation

Pose Purposeful Questions

- Q:** In Part A, how does the constant of variation k help determine the x - and y -values?
[Sample: The constant of variation k is the ratio of y to x . Therefore, any two values with a ratio equal to the same constant of variation satisfy the direct-variation relationship.]
- Q:** In Part B, why is it unimportant which pair of corresponding values you use to find k ?
[You are told that a direct variation relates the variables, so the ratio of y to x is the same for each pair of values.]

Try It! Answers

3. a. $y = -\frac{1}{6}x$
b. $y = 4x$

EXAMPLE 4 Use a Direct Variation Model

Elicit and Use Evidence of Student Thinking

- Q:** Which variable is replaced by k in the equation? [R]
- Q:** Why is it useful to first find the constant of variation?
[Sample: The constant of variation shows how voltage and current are related. Finding this variable first will provide an equation you can use to find the resistance for other voltage and current pairs.]

Try It! Answer

4. 7 hours



Examples are available at SavvasRealize.com.

EXAMPLE 3 Write a Direct Variation Equation

A. The value of y varies directly with x . When the value of x is 3, the value of y is -27 . What direct variation equation relates the values of x and y ?

Step 1 Find the constant of variation.

$y = kx$
 $-27 = k(3)$
 $-9 = k$

Substitute -27 for y and 3 for x . Then solve for k .

Step 2 Write the direct variation equation.

$y = -9x$

Substitute -9 for k .

The equation $y = -9x$ relates the values of x and y .

B. What direct variation equation relates x and y ?

x	-15	-9	-6	3	12
y	-5	-3	-2	1	4

Step 1 Find the constant of variation.

$y = kx$
 $-5 = k(-15)$
 $\frac{1}{3} = k$

Choose a pair of corresponding values. Then solve for k .

Step 2 Write the direct variation equation.

$y = \frac{1}{3}x$

Substitute $\frac{1}{3}$ for k .

The equation $y = \frac{1}{3}x$ relates the values of x and y .

Try It! 4. What direct variation equation relates the values of x and y ?

a. The value of y varies directly with x . When the value of x is -36 , the value of y is 6.

x	-2	2	3	5	6
y	-8	8	12	20	24

APPLICATION

EXAMPLE 4 Use a Direct Variation Model

Ohm's Law ($V = I \times R$) states that the electrical current I in amperes is directly proportional to the voltage V in volts. The constant of variation is resistance R measured in Ohms. Suppose a circuit consists of a 9-volt battery connected across a single resistor. The current in the circuit is 6 amperes. What current will this same resistor produce using a 12-volt battery?

Step 1 Find the constant of variation.

$V = IR$
 $9 = k(6)$
 $1.5 = k$

Write the equation where V is the voltage, I is the current, and R is the resistance.

Step 2 Use the direct variation equation with a voltage of 12 volts.

$12 = 1.5I$
 $8 = I$

Substitute 1.5 for k and 12 for V .

The resistor will produce a current of 8 amperes using a 12-volt battery.

Try It! 3. The number of parts built varies directly with the number of hours an assembly line operates. It takes 3 hours to build 84 parts. How many hours will it take to build 196 parts?

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Extend Student Thinking

USE WITH EXAMPLE 3 Have students explore direct variation involving fractions.

x	$-\frac{3}{4}$	$-\frac{1}{2}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{5}{6}$
y	9	6	-8	-9	-10

- Q:** Given that the values represent a direct variation, what is the value of k ? [$k = -12$]
- Q:** What direct-variation equation represents the data? [$y = -12x$]
- Q:** What is the x -value if $y = 1$? [$x = -\frac{1}{12}$]

Support Student Understanding

USE WITH EXAMPLE 1 Students may have difficulty connecting the algebraic and graphical representations of direct variation. Have them graph these data on separate graphs.

1.

x	1	2	4	6
y	3	6	10	16

2.

x	3	5	8	10
y	28	44	73	91

- Q:** How does the graph of the first relation show whether this is a direct variation?
[The graph is not a straight line, so it is not a direct variation.]
- Q:** How does the graph of the second relation show whether this is a direct variation?
[The graph looks like the points are on a straight line. To be sure, check $\frac{y}{x}$ for each point. The ratio is not constant, so it is not a direct variation.]



CONCEPT SUMMARY Direct Variation

Q: How can you determine whether the relationship between two variables is a direct variation?
[The relationship is a direct variation when the ratio of one variable to the other in an ordered pair is always equal to a constant of variation.]

Talk About Math Ideas

- Q:** How can you recognize that a real-world problem involves direct variation?
[Sample: The problem involves two variables that are related such that when one variable changes, the other variable changes proportionally.]
- Q:** Think about the relationships you discussed during the Explore & Share. Why is it useful to first represent a direct variation problem with an equation when solving it?
[Sample: You can substitute given information into the equation to solve the problem.]

Do You UNDERSTAND? | Do You KNOW HOW?

Common Error

Exercise 6 Students may think that a constant of variation is always the coefficient of x in a linear equation. Point out that k equals the coefficient of x only if the coefficient of y is 1. In the given equation, students should recognize that they should divide both sides of the equation by 4 to give y a coefficient of 1. This change will make the coefficient of x the fraction $\frac{12}{4}$. Remind students that it is helpful to reduce the fraction to its lowest form, 3, before using the equation to solve problems.

Answers

- Sample answer: Find the constant of variation. Then, write an equation in the form of $y = kx$, where k is the constant of variation. Use that equation to solve any related problems.
- Sample answer: You can write an equation in the form $y = kx$, where y is the distance traveled, x is the amount of gasoline used, and k is the constant representing the car's fuel efficiency (miles per gallon).
- Direct variation is a relation between two variables such that as one variable changes, the other changes proportionally.
- The quotient $\frac{y}{x}$ is not constant. It is different for each pair of x - and y -values.
- $\frac{1}{8}$
- Yes; 3
- $k = 60$ miles per hour; $d = 60t$; 210 miles



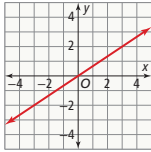
Concept Summary, Do You Understand, and Do You Know How are available at SavvasRealize.com.

CONCEPT SUMMARY Direct Variation

WORDS A direct variation is a relation between two variables such that as one variable increases, the other increases proportionally.

ALGEBRA $y = kx$, where $k \neq 0$

NUMBERS $y = \frac{3}{4}x$
The graph of a direct variation always passes through the origin.



Talk About Math Ideas How are direct variation equations used to solve problems in the real world?

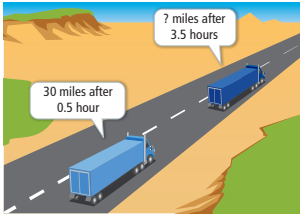
Do You UNDERSTAND?

- ESSENTIAL QUESTION** How do you write and solve equations involving direct variation?
- Explain Math Ideas** Explain why the distance traveled by a car and the gasoline used to travel that distance could represent a direct variation.
- Vocabulary** What is a direct variation?
- Error Analysis** Carmen said the table of values shown represents a direct variation. Explain why Carmen is mistaken.

x	1	2	3	4	8	16
y	24	12	8	6	3	2

Do You KNOW HOW?

- In a direct variation, $x = -8$ when $y = -\frac{1}{4}$. What is the value of y when $x = 4$?
- Does the equation $4y = 12x$ represent a direct variation? If so, what is the constant of variation?
- The distance a truck driver travels d , in miles, varies directly with the time t , in hours, spent driving. The driver travels 30 miles in 0.5 hour. What is the constant of variation, or speed, of the truck in miles per hour? Model the relationship between distance and time with an algebraic equation. If the truck driver travels for 3.5 hours at this rate, how many miles will she travel?





Practice

PRACTICE & PROBLEM SOLVING

Spaced and Interleaved Practice You may opt to assign the automatically scored lesson digital Practice and Problem Solving or Spiral Review.

Assignment Guide

On-Level	Advanced
9, 10, 11, 14, 15, 16, 17, 18, 19, 20	8, 12, 13, 21, 22, 23, 24, 25, 26, 27

Engage Through Student Choice

Go online for an assignment strategy.

Item Analysis

Example	Items	DOK	TEKS	
			Content	Process
1	8	3	A.2D	A.1F
	12	3	A.2D	
	14–15	2	A.2D	
2	9–11	2	A.2D	
	16	2	A.2D	
3	17–18	2	A.2D	
	19–20	2	A.2D	
4	13	3	A.2D	
	21	2	A.2D	
	22, 23, 25	2	A.2D	A.1A
	24	2	A.2D	A.1E
	26	2	A.2D	
	27	2	A.2D	

Answers

8. Sample answer: All the y -to- x ratios are equivalent, so the data represent a direct variation.
9. Yes; $p = 4s$, where p is the perimeter and s is the side length.
10. No; $A = \pi r^2 = \pi \left(\frac{d}{2}\right)^2 = \frac{\pi}{4}d^2$, where A is the area and d is the diameter.
11. No; the items you buy at the grocery store do not all cost the same amount, so the relationship is not direct.
12. A direct variation has a constant quotient $\frac{y}{x}$ only when $x \neq 0$.
13. Write the direct variation equation, $\ell = kw$.
Substitute 4.2 for ℓ and 3 for w to find k .
 $4.2 = 3k$; $k = 1.4$
Find ℓ when $w = 15$; $\ell = 1.4(15) = 21$.
Solve to find the perimeter.
 $P = 2\ell + 2w = 2(21) + 2(15) = 42 + 30 = 72$ in.



Practice & Problem Solving, Adaptive Practice, and Spiral Review are available at SavvasRealize.com.

PRACTICE & PROBLEM SOLVING

UNDERSTAND

8. **Analyze Math Relationships** Just from looking at the table of values, how can you determine that the data represent a direct variation?

x	-2	2	4	6	8	10
y	-6	6	12	18	24	30

Mathematical Connections Determine whether the two quantities vary directly. Explain your reasoning.

9. the perimeter of a square and the side length of the square
10. the area of a circle and the diameter of the circle
11. the amount of money you spend and the number of items you purchase at the grocery store
12. **Error Analysis** Describe and correct the error a student made in determining whether the table represents a direct variation.

x	y	$\frac{y}{x}$
0	0	undefined
1	3	3
2	6	3
3	9	3

Since 0 divided by 0 is undefined, there is no constant of variation, and the relation $y = 3x$ is not a direct variation.



13. **Higher Order Thinking** The length of the rectangle shown varies directly with its width. What is the perimeter of the rectangle if it is proportionally enlarged to have a width of 15 inches? Justify your answer.



$w = 3$ in.

$\ell = 4.2$ in.

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Lesson Help

PRACTICE

Does the table of values represent a direct variation? Explain. SEE EXAMPLE 1

14.

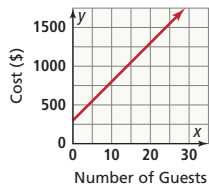
x	5	10	15	20	25
y	15	30	45	60	75

15.

x	-1	-2	-4	-5	-6
y	2	4	6	8	10

16. The graph shows the cost of catering a large function for a given number of guests. Does the graph represent a direct variation? Explain.

SEE EXAMPLE 2



Write a direct variation equation that relates the values of x and y . SEE EXAMPLE 3

17. The value of y varies directly with x . When the value of x is -4, the value of y is -80.
18. The value of y varies directly with x . When the value of x is 3, the value of y is 30.75.
- 19.
- | | | | | | |
|-----|-----|----|----|---|----|
| x | -12 | -8 | -4 | 0 | 8 |
| y | 3 | 2 | 1 | 0 | -2 |
- 20.
- | | | | | | |
|-----|---|-----|---|-----|----|
| x | 0 | 2 | 4 | 6 | 8 |
| y | 0 | 2.5 | 5 | 7.5 | 10 |
21. The area, in square feet, that paint can cover varies directly with the volume of the paint, in gallons. A 5-gallon paint can will cover 1,750 square feet. How many gallons of paint would be needed to cover an area of 4,200 square feet? SEE EXAMPLE 4



Practice

Answers

22. a. $\frac{2}{5}$
b. $F = \frac{2}{5}w$
c. 52 lb
23. $d = 1.5t$; 60 min
24. a. $m = 240c$
b. 240; 960; 1,920; 3840
25. a. $c = \frac{7}{36}s$
b. 7 cups
26. B
27. Part A $d = 66.5t$; 465.5 mi
Part B $c = kg$; $k = \$3.69$; \$55.35



Practice & Problem Solving, Adaptive Practice, and Spiral Review are available at SavvasRealize.com.

PRACTICE & PROBLEM SOLVING

APPLY

22. **Apply Math** The force you need to apply to a lever varies directly with the weight you want to lift. Suppose you can lift a 50-lb weight by applying 20 lb of force to a certain lever.
- a. What is the ratio of force to weight for the lever?
- b. Write an equation relating force and weight.
- c. What is the force you need to lift a friend who weighs 130 lb with this lever?
23. **Apply Math** The distance d , in meters, a submarine submerges after launch varies directly with the time t , in minutes, after launch. Twenty minutes after launch, the submarine is 30 meters below the ocean's surface. Write the equation of the direct variation. How many minutes will it take the submarine to submerge 90 meters at this same rate?
24. **Represent Math Ideas** The number of MP3 files that can be stored on a USB flash drive varies directly with the storage capacity of the flash drive. About 480 MP3 files can be stored on a flash drive that has a capacity of 2 gigabytes (GB).
- a. Write a direct variation equation that relates the number of MP3 files m that will fit on a flash drive with a capacity c in GB.
- b. Use the direct relationship to complete the table of values.
- | | | | | |
|--|---|---|---|----|
| Flash Drive Capacity in Gigabytes, c | 1 | 4 | 8 | 16 |
| Number of MP3 Files, m | ■ | ■ | ■ | ■ |
25. **Apply Math** The amount c , in cups, of mashed bananas in a banana bread recipes varies directly with the number of servings s . The number of cups of mashed bananas is $2\frac{1}{3}$ when the number of servings is 12.
- a. Write the equation of the direct variation.
- b. Find the number of cups of mashed bananas needed to make 36 servings of bread.

ASSESSMENT PRACTICE

26. **SAT** Suppose y varies directly as the square of x . If x is multiplied by 4, which of the following is true for the value of y ?
- Ⓐ It is multiplied by 4.
Ⓑ It is multiplied by 16.
Ⓒ It is multiplied by $\frac{1}{4}$.
Ⓓ It is multiplied by $\frac{1}{16}$.
27. **Performance Task** Suppose Cameron takes a road trip. He starts from his home in the suburbs of Cleveland, OH and travels to Pittsburgh, PA to visit his aunt and uncle. **A.2D**



Part A The distance Cameron drives from Cleveland to Pittsburgh is 133 miles. The trip takes him 2 hours. The distance d , in miles, that Cameron drives varies directly with the amount of time t , in hours, he spends driving. Write the equation of the direct variation. Use the given relationship and the equation to find the total number of miles Cameron would travel if he continues on for 5 more hours.

Part B The total cost c of the gasoline varies directly with the number of gallons purchased g . Write the equation of the direct variation. Cameron's car gets 25 miles per gallon on the highway. He drives 100 miles on the highway, which costs \$14.76 in gasoline. What is the value of k , the cost per gallon of the gasoline? If Cameron purchased 15 gallons of gasoline before his trip, how much did he spend?



QUICK CHECK

Use the Quick Check to assess students’ understanding of the mathematics in the lesson.

- You can assign the Quick Check to students to complete online where it will be automatically scored, and appropriate differentiated practice will be assigned based on student performance.
- You can download a printable copy from SavvasRealize.com.
- You can also find the Quick Check in the Assessment Sourcebook.
- You can download an editable version from SavvasRealize.com.

Item Analysis

Item	DOK	Skills Review and Practice	TEKS
1	2	F03, F04	A.2D
2	2	F03, F04	A.2D
3	2	F04	A.2D
4	2	F04	A.2D
5	2	F04, G07	A.2D

DIFFERENTIATED RESOURCES

Points	Practice
0–3 points	• Reteach to Build Understanding • Mathematical Literacy and Vocabulary • Lesson Student Tutorial videos
4–5 points	• Additional Practice • Enrichment



Quick Check is available at SavvasRealize.com.

ASSESSMENT RESOURCES

Name

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2-1 Quick Check

Direct Variation

1. Does the table of values represent a direct variation? Explain.

x	4	6	8	14
y	2	3	4	7

☒ A Yes because $\frac{y}{x}$ is constant.

☐ B Yes because $\frac{y}{x}$ is not constant.

☐ C No because $\frac{y}{x}$ is constant.

☐ D No because $\frac{y}{x}$ is not constant.

2. Which direct variation equation relates x and y?

x	-6	-2	3	9	12
y	18	6	-9	-27	-36

☐ A $y = \frac{1}{3}x$

☐ B $y = -\frac{1}{3}x$

☐ C $y = 3x$

☒ D $y = -3x$

3. The value of y varies directly with x. When the value of x is 12, the value of y is 3. What is the value of y when x is 6? **1.5**

4. The value of y varies directly with x. When the value of x is -2, the value of y is 10. What is the value of y when x is 8? **-40**

5. The perimeter of a square P varies directly with its side length s. When the side length of a square is 2.5 inches, the perimeter of the square is 10 inches. What is the side length of a square when its perimeter is 30 inches? **7.5** inches

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DIFFERENTIATED RESOURCES

Reteach to Build Understanding

Provides scaffolded reteaching for the key lesson concepts.

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2-1 Reteach to Build Understanding
Direct Variation

1. A direct variation is a relationship between two variables such that as one variable increases, the other variable increases proportionally. The equation $y = kx$, where k is the constant of variation, represents a direct variation. Complete the tables by solving for k , which is $\frac{y}{x}$. Then circle Yes or No below each table.

x	-5	-2	3	4	7
y	10	4	-6	-8	-14

Direct variation? **Yes** No

x	3	4	7	10	11
y	12	20	28	40	55

Direct variation? Yes **No**

2. Determine whether the equation represents a direct variation between x and y .

a. Fill in the blanks.

$-6x + 2y = 18$

$2y = 18 + \underline{6x}$ Add $6x$ to both sides.

$y = \underline{\frac{18}{2}} + \underline{\frac{6x}{2}}$ Divide both sides by 2.

$y = \underline{9} + 3x$ Simplify.

$y = \underline{3x} + 9$ Rearrange.

b. Is there a direct variation between x and y ? Explain.
There is not a direct variation between x and y because the equation cannot be written in the form $y = kx$.

3. The value of y varies directly with x . When the value of x is -8 , the value of y is -6 .

a. The constant of variation, k , for this relationship is $\underline{\frac{3}{4}}$.

b. An equation for the relationship is $y = \underline{(\frac{3}{4})x}$.

c. Complete the table with values in the relationship.

x	-8	-4	0	4	8
y	-6	-3	0	3	6

d. Graph the relationship.

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Additional Practice

Provides extra practice for each lesson.

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2-1 Additional Practice
Direct Variation

Determine whether each table of values represents a direct variation. Explain.

1.

x	2	4	8	9	13
y	6	12	24	27	39

Yes; $\frac{y}{x}$ is constant.

2.

x	1	3	5	7	9
y	3	6	9	12	15

No; $\frac{y}{x}$ is not constant.

3.

x	5	10	15	20	25
y	-2	-4	-6	-8	-10

Yes; $\frac{y}{x}$ is constant.

Determine whether each equation represents a direct variation. If it does, find the constant of variation.

4. $-2y = 8x$ **Yes; $k = -4$**

5. $2x - 5y = 0$ **Yes; $k = \frac{2}{5}$**

6. $3x + 4y = -5$ **No**

Suppose y varies directly with x . Write a direct variation equation that relates the values of x and y .

7. $y = 15$ when $x = 5$ **$y = 3x$**

8. $y = 2$ when $x = 9$ **$y = (\frac{2}{9})x$**

9. $y = -24$ when $x = 6$ **$y = -4x$**

Determine whether each situation represents a direct variation. Explain.

10. A company charges a flat fee of \$5.00 to ship a product plus an additional \$0.75 per pound that the product weighs.
No; The relationship cannot be expressed in the form $y = kx$.

11. A bottling machine in a factory fills 2,500 bottles per hour.
Yes; The relationship can be expressed in the form $y = kx$.

12. The perimeter of a square varies directly with the length of one side.

a. What is an equation that relates the perimeter, p , and length, l , of the side? **$p = 4l$**

b. Graph the equation.
Check students' graphs.

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Enrichment

Presents engaging problems and activities that extend the lesson concepts.

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2-1 Enrichment
Direct Variation

A rubber ball is dropped out a window. The height that the ball bounces varies with the height of the window. This relationship is modeled by the equation $y = 0.4x$, where x represents the height, in feet, from which the ball is dropped and y represents the height, in feet, the ball bounces.

1. Is there a direct variation between the height that the ball bounces and the height of the window? If so, what is the constant of variation? If not, explain why not?
Yes, there is a direct variation. The constant of variation is 0.4.

2. The ball is dropped from a height of 25 feet above the ground. How high does it bounce?
10 feet

3. The ball bounces 24 feet in the air. From what height above the ground was it dropped?
60 feet

4. Suppose a second ball is dropped and bounces 1.75 times as high as the first ball. How does the constant of variation of the relationship using the second ball compare to that of the first ball?
The constant of variation is 1.75 times the first.

5. What equation can be used to represent the height the second ball bounces?
 $y = 0.7x$

6. Graph both equations.

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Mathematical Literacy and Vocabulary

Helps emergent bilingual students develop and reinforce understanding of key terms and concepts.

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2-1 Mathematical Literacy and Vocabulary
Direct Variation

Suppose y varies directly with x , and $y = 36$ when $x = 9$. Latoya finds a direct variation equation relating x and y . She also finds the value of y when $x = 7$. The box below contains five phrases she uses for her thinking and five equations she uses for her written steps. In the table, the first column explains her thinking, and the second column shows her steps. Write the thinking and the steps in the correct order.

Think	Write
Start with the function form of a direct variation.	$y = 4x$
Divide each side by 9 to solve for k .	$36 = k(9)$
Find the value of y when $x = 7$.	$y = 4(7) = 28$
Substitute 9 for x and 36 for y .	$y = kx$
Write an equation. Substitute 4 for k in $y = kx$.	$4 = k$

Think	Write
First, she should start with the function form of a direct variation.	Step 1 $y = kx$
Second, she should substitute 9 for x and 36 for y .	Step 2 $36 = k(9)$
Next, she should divide each side by 9 to solve for k .	Step 3 $4 = k$
Then, she should write an equation. Substitute 4 for k in $y = kx$.	Step 4 $y = 4x$
Finally, she should find the value of y when $x = 7$.	Step 5 $y = 4(7) \text{ or } 28$

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Expand the Learning

You may choose to provide time in class to discuss innovations in the mathematics field by individuals highlighted in the **Amazing Contributions** Teacher Resource Original found on SavvasRealize.com. You may wish to provide the Teacher Resource Original found on SavvasRealize.com. Consider allowing time for students to research the person and their lasting, real-world impact and/or share responses to the THINK question.



TOPIC 2

Math Modeling in 3 Acts:
How Tall Is Tall?



Lesson Overview

Mathematics Objectives

- ✓ Use mathematical modeling to represent a problem situation and to propose a solution.
- ✓ Test and verify the appropriateness of their math models.
- ✓ Explain why the results from their mathematical models might not align exactly with the problem situation.

Language Objective

- ✓ Explain why the results from their mathematical models may differ from the exact answer.

Essential Understanding

Many real-world problem situations can be represented with a mathematical model, but that model might not represent the real-world situation exactly.

Earlier in this topic, students:

- Interpreted equations.

In this lesson, students:

- Develop a mathematical model to represent and propose a solution to a problem situation involving linear equations and functions.

Later in this course, students will:

- Identify parallel and perpendicular lines.
- Refine their mathematical modeling skills.

This mathematical modeling lesson focuses on application of both math content and math processes.

- Students draw on their understanding of concepts related to linear equations to develop a representative model.
- Students apply their mathematical model to test and validate its applicability to similar problem situations.

MATH
MODELING
IN 3 ACTS

Mathematical Process
Standards A.1A
Apply mathematics to
problems arising in everyday
life, society, and the workplace.
TEKS
A.5B

ACT 1

BUILD G.R.I.T. Grow
Confidence
Develop confidence. As you work
on your conjecture and reason
with your peers, continue to think
positively about your ability to
come up with responses that
make sense.

TOPIC 2 Math Modeling in 3 Acts 81

How Tall Is Tall?

The world's tallest person in recorded history was Robert Wadlow. He was 8 feet 11.1 inches tall! Only 5% of the world's population is 6 feet 3 inches or taller. What percent of the population would you guess is 7 feet or taller?

We usually use standard units, such as feet and inches or centimeters, to measure length or height. Did you ever wonder why? In the Math Modeling in 3 Acts lesson you'll consider some interesting alternatives.

ACT 1 Identify the Problem

1. What is the first question that comes to mind after watching the video?
2. Write down the main question you will answer about what you saw in the video.
3. Make an initial conjecture that answers this main question.
4. Explain how you arrived at your conjecture.
5. Write a number that you know is too small.
6. Write a number that you know is too large.
7. What information will be useful to know to answer the main question? How can you get it? How will you use that information?

ACT 2 Develop a Model

8. Use the math that you have learned in this Topic to refine your conjecture.

ACT 3 Interpret the Results

9. Is your refined conjecture between the highs and lows you set up earlier?
10. Did your refined conjecture match the actual answer exactly? If not, what might explain the difference?

Student Companion

Students can do their work for the task on their *Student Companion* which you can print from **SavvasRealize.com**.

Texas Essential Knowledge and Skills (TEKS)

Mathematical Process Standards

The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

A.1A apply mathematics to problems arising in everyday life, society, and the workplace.

Linear functions, equations, and inequalities: The student applies the mathematical process standards to solve, with and without technology, linear equations and evaluate the reasonableness of their solutions. The student is expected to:

A.5B solve linear inequalities in one variable, including those for which the application of the distributive property is necessary and for which variables are included on both sides.

TOPIC 2 | 81 | Math Modeling in 3 Acts

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How Tall Is Tall?

In this mathematical modeling task, students explore and apply linear functions. Students will be presented with a situation in which height is measured in unconventional ways. They will be tasked with creating and using a linear model to measure height in terms of foam cups. To do so, they apply concepts that they study in Topic 2.

ACT 1
 The Hook

Play the video. The video shows the height of a basketball player in terms of other objects. A stack of cups starts to build next to him.

After the question brainstorming, present to students the Main Question they will be tasked with answering. Remind students to write down their questions and conjectures.

MAIN QUESTION

What is the basketball player’s height in foam cups?



ACT 2
 Modeling With Math

Think about the task. Ask students to speculate how they could figure out how tall the player is in cups. Then let them think about what information they would need to find the answer.

Reveal the information. Use the video to provide data about smaller stacks of cups. Suggest that students record this information, possibly in a table. Then ask them to look for patterns and think about ways they can use the data to answer the Main Question. How could they represent the relationship between the number of cups and the height of the stack? How could they use that representation to generate more data?

What’s the connection? Give students time to struggle as they think about how their ideas are connected to what they learned in this topic about using a linear function to represent a situation.

INTERESTING MOMENTS WITH STUDENTS

Students may recognize a pattern and then struggle. Students may see that you add 0.6 inches for each additional cup in the stack, but the prospect of adding until they reach 80 inches sounds miserable! Help them turn a sequence into a linear function.

Students may divide the height of the person by the height of one cup. This assumes that two cups will be twice the height of one cup. You can physically demonstrate what their model means (the cups would hover over each other), or you can replay Act Two to show how the cups are stacked inside one another.

Necessary Information

Player height
 80 in. (including shoes)

Foam cups
 height of the lip: 0.6 inches
 height of the base: 4 inches
 All cups are the same size.



ACT 3 The Solution

Play the video. The final video shows all of the cups falling into place. The player’s height, in foam cups, is revealed. Offer praise to the students who had the closest mathematical answer and closest guess.

MAIN QUESTION ANSWER

The player’s height is 130 cups.

Do the “post-game” analysis. It turns out that the more cups you stack on top of each other, the more the weight of all the cups on top compresses the cups near the bottom of the stack. While the calculated solution is approximately 127 cups, the actual answer is greater—130 cups.

Now that the solution has been revealed and discussed, ask students to analyze and evaluate the efficiency of the approach they chose to solve the problem.

ONE POSSIBLE SOLUTION

There is a linear relationship between the number of foam cups in the stack and the height of the stack. The equation $h = 4 + 0.6n$, where h is the height of the stack and n is the number of cups, can represent the situation.

Since the player’s height is 80 inches, solve the equation $80 = 4 + 0.6n$.

$$80 = 4 + 0.6n$$

$$76 = 0.6n$$

$$127 \approx n$$

There are approximately 127 cups in the stack.

INTERESTING MOMENTS WITH STUDENTS

Students may ask about 0 cups. Our function says that 0 cups have a height of 4 inches. This is where you can talk about domain. 0 is not in the domain of our function, nor are some rational numbers like $3\frac{1}{2}$, because they do not make real-world sense. Therefore, the domain is limited to the natural numbers.

CREATE A PROBLEM

As students finish the original task, ask them to construct their own problem to find the height of an object that is a certain number of foam cups tall. Challenge students to use the linear relationship they defined in the task to determine the height in feet and then choose an appropriate object.

